

Polarimetric glucose sensing using Brewster-reflection off of eye lens: Theoretical analysis

Stefan Böckle,^a Luigi Rovati,^a and Rafat R. Ansari^{b*}

^aDepartment of Electronics for the Automation
University of Brescia, via Branze 38, I-25123 Brescia (Italy)

^bNASA Glenn Research Center
Mail Stop 333-1, 21000 Brookpark Road, Cleveland, Ohio 44135

ABSTRACT

An important task of in vivo polarimetric glucose sensing is to find an appropriate way to optically access the aqueous humor of the human eye. In this paper two different approaches are analyzed theoretically and applied to the eye model of Le Grand. First approach is the tangential path of Coté *et al.*, and the second is a new scheme of this paper of applying Brewster reflection off the eye lens.

Keywords: non-invasive glucose sensor, polarimetry, diabetes

1. INTRODUCTION

The non-invasive detection of blood glucose levels in humans is an ambitious goal for managing diabetes. Diabetes is the fourth leading cause of mortality in the United States. Diabetes can lead to severe complications over time. These can include blindness, renal and cardiovascular diseases, peripheral neuropathy associated with limb. The poor blood circulation in lower extremities of the body can lead to gangrene and subsequent amputation. Over 100 million diabetics world wide could greatly benefit by better managing their diabetes if their glucose levels can be monitored frequently and non-invasively without the pain and inconvenience of presently used fingerstick blood tests or implants.

Several non-invasive optical techniques have the potential of avoiding the disadvantages of the standard fingerstick method, and thus enable a more frequent determination of glucose levels. These methods are primarily based upon the measurement of the glucose concentration in the aqueous humor using polarimetry [2-3], dermal, epidermal, interstitial fluid, and sweat using absorption spectroscopy. Test sites being explored include eye, fingertips, cuticle, finger web, forearm and ear lobe. However, many hurdles remain before these products reach the commercial marketplace.

Glucose concentration in the aqueous humor closely mimics glucose levels in the blood [4-5]. Polarimetric techniques use the property of glucose as an optically active analyte, that rotates the plane of polarization of incident linearly polarized light in accordance with the law [2-5]:

$$\alpha = [\alpha]_{\lambda, pH}^T \cdot l \cdot c, \quad (1)$$

where $[\alpha]_{\lambda, pH}^T$ is the specific rotation of glucose, l the optical pathlength through the sample and c the glucose concentration in the sample. Thus, knowing the pathlength l inside the sample, the glucose concentration c can be calculated by observing the rotation α , that the glucose molecules induce to linearly polarized light. Previously, such a measurement was performed by passing a beam of linearly polarized light crosswise (tangentially) through the aqueous and by observing α on the other end [1-2]. To best of our knowledge, to this date, no commercial device exploiting this simple optical working principle has been realized. We believe the major reason for this is that the proposed optical path plays a critical role due to the limits imposed by the shape of the cornea.

* Corresponding author

In this paper two different approaches for optically accessing the aqueous humor are analyzed theoretically with respect to their geometrical properties of light propagation and applying them to the eye model of Le Grand [6]. First, the tangential optical path proposed by Coté *et al.* [1-2] is investigated. Second, the new optical scheme using Brewster reflection off the lens will be introduced.

2. THEORETICAL ANALYSIS

The beam path of light propagation inside the eye is calculated applying the Snell's law of refraction for transitions between the media of different refractive indices. As the ocular media are assumed to be homogeneous and isotropic the calculation of the beam path is reduced to simple geometric considerations.

2.1. Eye-model

In the present investigation the eye model of Le Grand is applied [6]. The essential properties of this model are given in Table 1 and Fig. 1.

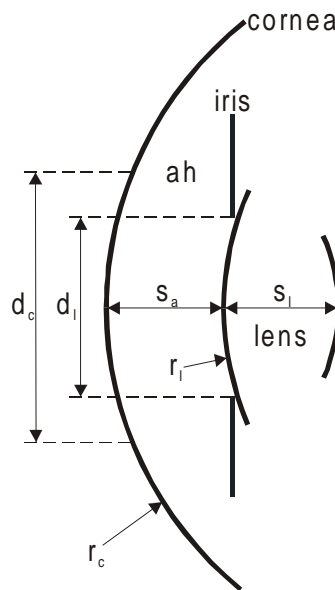


Figure 1: Eye-model of Le Grand; ah=aqueous humor.

Table 1: Parameters of Le Grand eye-model.

Medium	Front diameter [mm]	Radius [mm]	Spacing [mm]	Refractive index
Cornea	$d_c=10$	$r_c=8$	—	$n_c=1.336$
Aqueous	—	—	$s_a=5.7763-6.3740$	$n_h=1.336$
Lens (ant.)	$d_l=2-8$	$r_l=10.2$	$s_l=4$	$n_l=1.4208$

As shown in Fig. 1, in this model the optical system of the eye consists of three centered, spherical refracting surfaces. Thus, the optical system shows rotational symmetry. Cornea and aqueous humor are assumed to have equal refractive indices with an effective value that accounts for this simplification. Therefore, there is only one effective refracting surface between air and aqueous humor. All ocular media are assumed to be homogeneous and isotropic. Thus, the linear birefringence of the cornea will be left out of consideration. This means no limitation for the validity of the present study as this effect can be discriminated from the circular birefringence of the glucose through an accurate analysis of the polarization state of the output beam [7]. Moreover, the shell structure of the lens is neglected and an average refractive index is considered. These assumptions provide us with a relatively simple optical system for the calculations.

2.2. Tangential path

In this approach, incident linearly polarized light is passed tangentially through the aqueous humor, as is shown in Fig. 2.

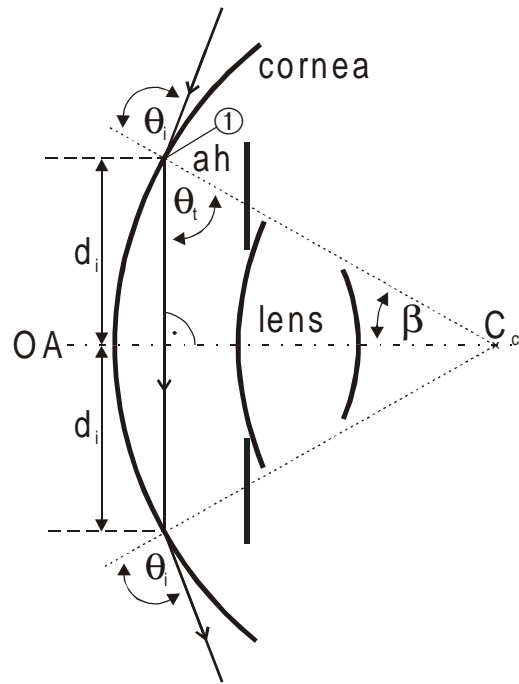


Figure 2: Tangential optical path: The incident light (1) is linearly polarized. Abbreviations: OA=optical axis, ah=aqueous humor, C_c =center point of sphere of cornea.

We calculate the minimum distance of the incidence point (1) from the optical axis (OA), d_i , that is necessary to achieve the tangential path. The incident beam at the air-aqueous humor interface has to meet the condition to be smaller than 90° with respect to the normal to the surface at the intersection point:

$$\theta_i < 90^\circ \quad (2)$$

Although the condition $\theta_i=90^\circ$ is not practicable because the beam would not enter the eye we use this condition as the limit for the applicability of the present approach.

Applying the Snell's law for this boundary condition we get the refraction angle of the transmitted beam:

$$\theta_t = \arcsin\left(\frac{1}{n_h}\right) \quad (3)$$

with: $\theta_i = 90^\circ$,

where the refractive index of air is set to unity.

The minimum distance d_i of the incidence point from the optical axis is then given by:

$$d_i = r_c \cdot \sin(\beta), \quad (4)$$

where β is defined as:

$$\beta = 90^\circ - \theta_i. \quad (5)$$

Thus, using Eq. (3) and the values given in Table 1, d_i can be calculated to:

$$d_i = r_c \cdot \sin(90^\circ - \theta_i) = 5.305 \text{ mm}. \quad (6)$$

As θ_i has to meet $\theta_i < 90^\circ$, it follows due to the Snell's law that θ_i amounts to $\theta_i < \arcsin(1/n_h)$ and thus, the value for the distance of the incidence point from the optical axis yields $d_i > 5.3 \text{ mm}$.

However, as reported in Table 1, the average human eye exposes only a diameter of $d_c = 10 \text{ mm}$ of its surface for optical access [6]. Thus, for this eye-model, the tangential path is theoretically not achievable.

2.3. Brewster reflection off the lens

As shown in Fig. 3, we propose a novel approach to access the aqueous humor of the eye. Part of the incident beam is reflected off the lens at the Brewster's angle yielding a purely linearly polarized reflected beam (2), orientated perpendicularly to the plane of refraction (paper plane). On its way out of the eye the polarization state of this beam is rotated by the glucose in the aqueous humor and thus carries the concentration information. Notice, that the propagation path shows symmetry with respect to OA.

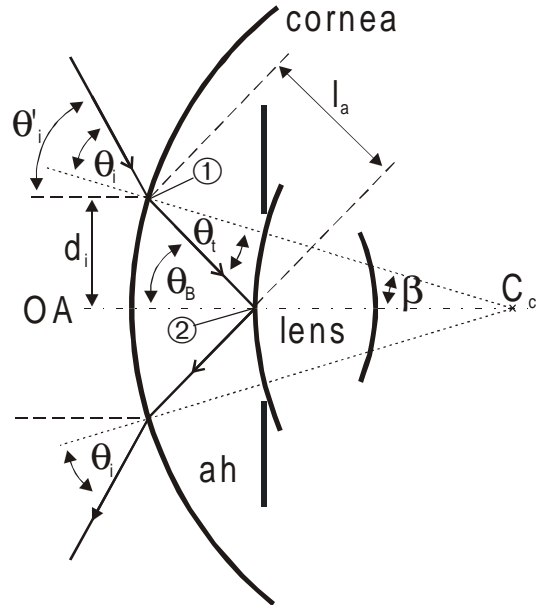


Figure 3: Brewster reflection off the lens. Abbreviations: OA=optical axis, ah=aqueous humor, C_c =center point of sphere of cornea.

Referring to Figs. 1 and 3 we consider the triangle given by the corner points at position (1), (2) and C_c . The cosine rule, applied to this triangle, delivers a second order equation for l_a , which can be solved as:

$$l_a = -(r_c - s_a)\cos(\theta_B) + \sqrt{(r_c - s_a)^2 \cos^2(\theta_B) - s_a^2 + 2s_a r_c} = 6.496 \text{ mm} . \quad (7)$$

This leads directly to:

$$d_i = l_a \cdot \sin(\theta_B) = 4.73 \text{ mm} . \quad (8)$$

Applying simple geometrical relations, the entrance angle θ_i^I can be calculated by the following steps:

$$\begin{aligned} \beta &= \arcsin\left(\frac{d_i}{r_c}\right) = 36.27^\circ \\ \theta_t &= \theta_B - \beta = 10.49^\circ \\ \theta_i &= \arcsin(n_h \cdot \sin(\theta_t)) = 14.08^\circ \\ \theta_i^I &= \theta_i + \beta = 50.35^\circ . \end{aligned} \quad (9)$$

In contrast to the approach discussed in 2.2, within the limits of this eye model, the anterior diameter d_c of the cornea is sufficient to accept a beam entering at distance d_i from the ocular axis. The entrance angle θ_i^I is, as well, not critical to be achieved.

3. CONCLUSIONS

A detailed geometrical analysis of two different schemes for optically accessing the aqueous humor of the human eye for in vivo polarimetric glucose measurements is presented. The newly proposed optical path using Brewster reflection off the lens is demonstrated to be theoretically applicable. In conclusion, Brewster reflection off the lens has the potential to be the future instrumental arrangement for in vivo polarimetric glucose sensing. This work is in progress.

4. REFERENCES

1. G.L. Coté, M.D. Fox, R.B. Northrop, "Noninvasive Optical Polarimetric Glucose Sensing using a True Phase Measurement Technique," IEEE Transactions on Biomedical Engineering, Vol. 39, No. 7, pp. 752–756, 1992.
2. B.D. Cameron, G.L. Coté, "Noninvasive Glucose Sensing Utilizing a Digital Closed-Loop Polarimetric Approach," IEEE Transactions on Biomedical Engineering, Vol. 44, No. 12, pp. 1221–1227, 1997.
3. D.P. Hutchinson, "Personal glucose monitor," United States Patent 4 901 728, 1990.
4. R.J. McNichols and G.L. Coté, "Optical Glucose Sensing in Biological Fluids: An Overview," J. Biomedical Optics, 5(1):5–16. Review, Jan. 2000.
5. C.A. Browne and F.W. Zerban, "Physical and Chemical Methods of Sugar Analysis," New York, London: Chapman & Hill, 1941.
6. Y. Le Grand and S.G. El Hage, Physiological Optics (Springer-Verlag, Berlin, 1980).
7. L. Rovati and R. Ansari, Optics letter, submitted 2001.